DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

357. Proposed by V. M. SPUNAR, M. and E. E., Chicago, Ill.

Solve the system

$$\sqrt{(x^2+a^2+b^2+c^2)}=\sqrt{(y^2+b^2+c^2)}+\sqrt{(z^2+b^2+c^2)},$$
$$\sqrt{(y^2+a^2+b^2+c^2)}=\sqrt{(x^2+a^2+c^2)}+\sqrt{(z^2+a^2+c^2)},$$
$$\sqrt{(z^2+a^2+b^2+c^2)}=\sqrt{(x^2+a^2+b^2)}+\sqrt{(y^2+a^2+b^2)}.$$

Solution by B. F. FINKEL, Ph. D., Drury College.

Transposing the first term of the second member of the first equation, squaring, collecting, and transposing, we have

$$x^2+y^2-z^2+s^2=2\sqrt{(x^2+s^2)}\sqrt{(y^2+s^2-a^2)}\ldots(1),$$

where $s^2=a^2+b^2+c^2$. Similarly, we obtain from the second equation,

$$x^2+y^2-z^2+s^2=2\sqrt{(y^2+s^2)}\sqrt{(x^2+s^2-b^2)}\ldots(2),$$

and from the third,

$$x^2+z^2-y^2+s^2=2\sqrt{(z^2+s^2)}\sqrt{(x^2+s^2-c^2)}\ldots(3).$$

By transposing the second term of the first equation, squaring, collecting, and retransposing, we get

$$x^2+z^2-y^2+s^2=2\sqrt{(x^2+s^2)}\sqrt{(z^2+s^2-a^2)}\ldots(4).$$

From (1) and (2) we have,

$$\sqrt{(x^2+s^2)}\sqrt{(y^2+s^2-a^2)}=\sqrt{(y^2+s^2}\sqrt{(x^2+s^2-b^2)};$$

whence, by squaring and dividing by $(x^2+s^2)(y^2+s^2)$, we get

$$\frac{y^2+s^2-a^2}{y^2+s^2}=\frac{x^2+s^2-b^2}{x^2+s^2}.$$

Hence, $\dfrac{a^2}{y^2+s^2}=\dfrac{b^2}{x^2+s^2}$, or $y^2+s^2=\dfrac{a^2}{b^2}(x^2+s^2)$.

From (3) and (4), we obtain, in like manner,

$$z^2+s^2=\frac{a^2}{c^2}(x^2+s^2).$$

Substituting these values of z^2+s^2 and y^2+s^2 in the first equation, we have

$$\sqrt{(x^2+s^2)}=\sqrt{\frac{a^2}{b^2}(x^2+s^2)-a^2}+\sqrt{\frac{a^2}{c^2}(x^2+s^2)-a^2};$$

whence, $bc\sqrt{(x^2+s^2)}=ab\sqrt{(x^2+s^2-b^2)}+ab\sqrt{(x^2+s^2-c^2)}$.

Squaring,

$$b^2c^2(x^2+s^2)=a^2c^2(x^2+s^2-b^2)+2a^2bc\sqrt{(x^2+s^2-b^2)}\sqrt{(x^2+s^2-c^2)}+a^2b^2(x^2+s^2-c^2);$$

transposing, and combining,

$$(b^2c^2-a^2c^2-a^2b^2)(x^2+s^2)+2a^2b^2c^2=2a^2b^2c^2\sqrt{(x^2+s^2-b^2)}\sqrt{(x^2+s^2-c^2)}.$$

Squaring both members of this equation, and rearranging terms, we have

$$[(b^2c^2-a^2c^2-a^2b^2)^2-4a^4b^2c^2](x^2+s^2)+4a^2b^4c^4(x^2+s^2)=0.$$

$\therefore x^2+s^2=0$, or $x^2+s^2=$

$$\frac{4a^2b^4c^4}{4a^4b^2c^2-(b^2c^2-a^2c^2-a^2b^2)^2}$$

$$=\frac{4a^2b^4c^4}{(b^2c^2-a^2c^2-a^2b^2+2a^2bc)(-b^2c^2+a^2c^2+a^2b^2+2a^2bc)}$$

$$=\frac{4a^2b^4c^4}{[b^2c^2-a^2(b-c)^2][a^2(b+c)^2-b^2c^2)]}$$

$$=\frac{4a^2b^4c^4}{(bc+ab-ac)(bc-ab+ac)(ab+ac+bc)(ab+ac-bc)}$$

$=\dfrac{4a^2b^4c^4}{\triangle}$, where $\triangle$ is the denominator of the above fraction.

$$y^2+s^2=\frac{4a^4b^2c^4}{\triangle}, \text{ and } z^2+s^2=\frac{4a^4b^4c^2}{\triangle}.$$

$$\therefore x=\pm\left(\frac{4a^4b^2c^4-s^2\triangle}{\triangle}\right)^{\frac{1}{2}},\ y=\pm\left(\frac{4a^4b^2c^4-s^2\triangle}{\triangle}\right)^{\frac{1}{2}}, \text{ and}$$

$$z=\pm\left(\frac{4a^4b^4c^2-s^2\triangle}{\triangle}\right)^{\frac{1}{2}}.$$

$x^2+s^2=0$ is not admissible.

PROBLEMS FOR SOLUTION.

ALGEBRA.

363. Proposed by E. B. ESCOTT, Ann Arbor, Mich.

(a) If a and n be positive integers, the integral part of $[a+\surd(a^2-1)]^n$ is odd.

(b) If a and n be positive integers, the integral part of $[\surd(a^2+1)+a]^n$ is odd when n is even and even when n is odd. [From Todhunter's *Algebra*, p. 353].

364. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

The English physicist, Hooke, published the discovery contained in the Latin sentence, "Ut tensio sic vis" by the cypher *cciiinosssttuv*. Preserving the lexicographical order, find which permutation, taking all letters, the Latin sentence is from the cypher.

365. Proposed by C. N. SCHMALL, New York City.

In still water, a steam tug goes 6 miles an hour less when towing a barge than when alone. Having drawn the barge 30 miles up a stream, whose current runs 1 mile an hour, it returns alone and completes the journey in 12 8/11 hours. Find the rate of the tug in still water.

GEOMETRY.

396. Proposed by DANIEL KRETH, Oxford, Iowa.

In the triangle ABC, $AB=214$, $BC=263$, and $AC=405$. A point P is situated in the same horizontal plane; angle $BPA=13°\ 30'$ and angle $BPC=29°\ 50'$. Find the distances, AP, BP, and CP.